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MHD Free Convection from an Isothermal Truncated Cone with Variable Viscosity and Internal Heat Generation (Absorption)

A.H.Srinivasa*, A.T.Eswara**
*Department of Mathematics, maharaja Institute of Technology Mysore, Belawadi – 571438, India
** GSSS Institute of Engineering and Technology for Women, K.R.S. Road, Mysuru – 570016, India

ABSTRACT
This paper presents a study of MHD free convection flow of an electrically conducting incompressible fluid with variable viscosity about an isothermal truncated cone in the presence of heat generation or absorption. The fluid viscosity is assumed to vary as a inverse linear function of temperature. The non-linear coupled partial differential equations governing the flow and heat transfer have been solved numerically by using an implicit finite - difference scheme along with quasilinearization technique. The non-similar solutions have been obtained for the problem, overcoming numerical difficulties near the leading edge and in the downstream regime. Results indicate that skin friction and heat transfer are strongly affected by, both, viscosity-variation parameter and magnetic field. In fact, the transverse magnetic field influences the momentum and thermal fields, considerably. Further, skin friction is found to decrease and heat transfer increases near the leading edge. Also, it is found that the direction of heat transfer gets reversed during heat generation.
Keywords - Variable viscosity, MHD, Free convection, Skin friction, Heat transfer, Truncated cone.

I. INTRODUCTION

Natural or free convection is a mechanism, or type of heat transport, in which the fluid motion is not generated by any external source (like a pump, fan, suction device, etc.) but only by density differences in the fluid occurring due to temperature gradients. Natural convection is frequently encountered in our environment and engineering devices. A very common industrial application of natural convection is free air cooling without the aid of fans: this can happen on small scales (computer chips) to large scale process equipments.

The problem of natural convection flow over the frustum of a cone has been investigated by several authors [1-6] and, in all the above studies the viscosity of the fluid had been assumed to be constant. However, it is known that viscosity can change significantly with temperature[7-12]. Recently, Hossain and Kabir [13] have investigated the natural convection flow from a vertical wavy surface with viscosity proportional to an inverse linear of temperature. There has been a great interest in the study of magneto hydrodynamic (MHD) flow and heat transfer in any medium due to the effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow has attracted the interest of many researchers [14-16] due to its applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions. Of late, Srinivasa et.al [17] have considered the effect of variable viscosity on MHD free convection from an isothermal truncated cone. The present investigation extends the study of [17] to include the effects of internal heat generation and absorption.

II. MATHEMATICAL FORMULATION

We consider the steady, two-dimensional laminar natural convection flow of a viscous incompressible fluid about a truncated cone along with an applied magnetic field. Figure 1 shows the flow model and physical coordinate system. The origin of the coordinate system is placed at the vertex of the full cone, where x is the coordinate along the surface of the cone measured from the origin, and y is the coordinate normal to the surface. A transverse magnetic field of strength B_0 is applied in the direction normal to the surface of the truncated cone and it is assumed that magnetic Reynolds number is small, so that the induced magnetic field can be neglected. The boundary layer is assumed to develop at the leading edge of the truncated cone ($x = x_0$) which implies that the temperature at the circular base is assumed to be the same as the ambient temperature T_∞. The temperature of the surface of the cone T_w is uniform and higher than the free stream temperature T_∞ ($T_w > T_\infty$). As stated in the introduction, property variations with temperature are limited to and viscosity. However, variations in the density are taken into account only in so far as its effect on the buoyancy term in the momentum equation is concerned (Boussinesq approximation).

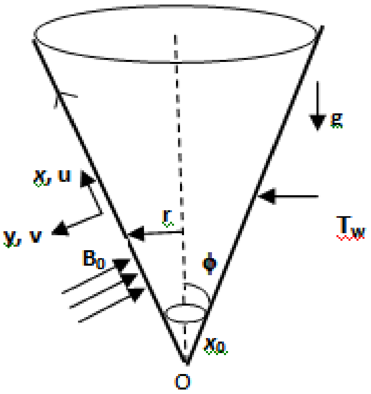


Figure. 1 The Geometry and the coordinate system

Under the above assumptions, the two-dimensional MHD boundary layer equations for natural convective flow of the electrically conducting fluid over a truncated cone, valid in the domain $x_0 \leq x \leq \infty$, are as follows:

$$\frac{\partial (ru)}{\partial x}+\frac{\partial (rv)}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\cos\phi(T-T_\infty)+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2(x)}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty) \tag{3}$$

The boundary conditions to be satisfied by the above equations are given by

$$\left.\begin{array}{l} u=0,\ v=0,\ T=T_w \ \text{ at } \ y=0 \\ u=0,\ T=T_\infty \ \text{ as } \ y\to\infty \end{array}\right\} \tag{4}$$

In the present investigation, a semi-empirical formula

$$\frac{\mu}{\mu_\infty}=\frac{1}{1+\gamma(T-T_\infty)} \tag{5}$$

for the viscosity of the form as developed by Ling and Dybbs[18], has been adopted, where μ_∞ is the viscosity of the ambient fluid and γ is a constant.

We have assumed the boundary layer to be sufficiently thin in comparison with the local radius of the truncated cone. The local radius to a point in the boundary layer can be replaced by the radius of the truncated cone r, $r = x\sin\phi$, where ϕ is semi vertical angle of the cone.

Introducing the following transformations:

$$\psi=\nu r\left(Gr_{x^*}^{1/4}\right)f(\xi,\eta),\quad \eta=\frac{y}{x^*}\left(Gr_{x^*}^{1/4}\right), \tag{6}$$

$$f'=\frac{\partial f}{\partial \eta},\quad \Pr=\frac{\nu}{\alpha}\ \xi=\frac{x^*}{x_0}=\frac{x-x_0}{x_0},$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\quad M=\frac{\sigma B_0^2(x)x^{*2}}{\rho Gr_{x^*}^{1/2}},$$

$$Q=\frac{x^{*2}Q_0}{\rho Gr_{x^*}^{1/2}c_p},\ ru=\frac{\partial\psi}{\partial y}\ \&\ =-\frac{\partial\psi}{\partial x},$$

$$Gr_{x^*}=\frac{g\beta\cos\phi(T_w-T_\infty)x^{*3}}{\nu^2}$$

to equations (1)-(3), we see that the continuity equation (1) is identically satisfied and the Eqs. (2) and (3) reduce, respectively, to

$$(1+\varepsilon\theta)f'''+(1+\varepsilon\theta)^2\left[\left(\frac{3}{4}+\frac{\xi}{1+\xi}\right)ff''-\frac{1}{2}f'^2+\theta-Mf'\right]-\varepsilon\theta'f''=(1+\varepsilon\theta)^2\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \tag{7}$$

$$\Pr^{-1}\theta''+\left(\frac{3}{4}+\frac{\xi}{1+\xi}\right)f\theta'+Q\theta=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \tag{8}$$

where ε (= $(T_w - T_\infty)\gamma$) is termed as the viscosity variation parameter, which is positive for heated surface and negative for a cooled surface.

Here ν $(=\mu_\infty/\rho)$ is the free stream kinematic viscosity, ψ and f is dimensional and dimensionless stream function, respectively, η is the pseudo similarity variable and θ is the dimensionless temperature of the fluid in the boundary layer region. where u, v are the fluid velocity components in the x- and y-direction, respectively, g is the gravitational acceleration, β is the coefficient of thermal expansion, T is the temperature inside the boundary layer, α is the thermal diffusivity, ρ is the fluid density, μ is the dynamic viscosity of the fluid, M non-dimensional magnetic parameter, Pr is Prandtl number, Gr_x is local Grashof number, ε is viscosity variation parameter, x streamwise coordinate, x^* distance measured from the leading edge of the truncated cone, ξ dimensionless distance.

The heat generation or absorption parameter Q appearing in Eqn. (8) is the non-dimensional parameter based on the amount of heat generated or absorbed per unit volume given by $Q_0(T-T_\infty)$, with Q being constant coefficient that may take either positive or negative values. The source term represents the heat generation that is distributed everywhere when Q is positive ($Q>0$) and the heat absorption when Q is negative ($Q<0$); Q is zero, in case no heat generation or absorption.

The boundary conditions for the above non dimensional equations (7)-(8) are given by

$$\left.\begin{array}{l} f(\xi,0)=0,\ f'(\xi,0)=0,\ \theta(\xi,0)=1,\ \text{at}\ \ \eta\to 0 \\ f'(\xi,\infty)=0,\ \theta(\xi,\infty)=0 \quad \text{as}\ \ \eta\to\infty \end{array}\right\} \tag{9}$$

In practical applications, the physical quantities of principle interest are the shearing stress τ_w and the rate of heat transfer in terms of the skin friction coefficient (C_f) and Nusselt number (Nu), respectively, which are written as

$$C_f Gr_{x^*}^{1/4}=\frac{2x^{*2}\tau_w}{Gr_{x^*}\rho}=\frac{2x^{*2}\left(\mu\frac{\partial u}{\partial y}\right)_w}{\rho Gr_{x^*}}=\left(\frac{2}{1+\varepsilon}\right)f''(\xi,0) \tag{10}$$

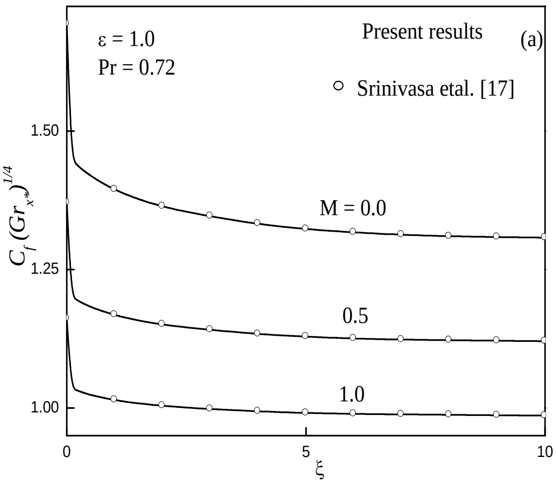
$$Nu_{x^*}Gr_{x^*}^{-1/4} = -\frac{x^*\left(\frac{\partial T}{\partial y}\right)_w}{\Delta T_w} = -\theta'(\xi,0) \qquad (11)$$

III. RESULTS AND DISCUSSION

The system of coupled, non linear partial differential equations (7) and (8) along with the boundary conditions (9) using the relations (10) - (11) has been solved numerically employing an implicit finite difference scheme along with quasilinearization technique. Since the method is described in great detail in Ref [19], its description is omitted here for the sake of brevity. In order to assess the accuracy of our numerical method, our results are found to be in good agreement with those of [17], correct to four decimal places of accuracy.

The numerical results are obtained for various values of magnetic field parameter M ($0 \le M \le 1.0$), viscosity variation parameter ε (=0, 0.5, 1.0) and presented graphically in Fig. 2 – 7.

The skin friction coefficient $[C_f(Gr_{x^*})^{1/4}]$ and heat transfer coefficient $[Nu(Gr_{x^*})^{-1/4}]$ for various values of magnetic field M (= 0.0, 0.5, 1.0) and for viscosity variation parameter ε = 1.0 and Pr = 0.72 along the streamwise coordinate (ξ) is presented in Figures 2(a) & 2(b), respectively. It is evident that $C_f(Gr_{x^*})^{1/4}$ and $Nu(Gr_{x^*})^{-1/4}$ found to decrease with increase of M. Also, $C_f(Gr_{x^*})^{1/4}$ is observed to decrease near the leading edge (ξ = 0), while $Nu(Gr_{x^*})^{-1/4}$ exhibits an increasing trend near ξ = 0. The percentage of decrease in $C_f(Gr_{x^*})^{1/4}$ is 33.2% near ξ = 5.0, when M is increasing from M = 0.0 to M = 1.0. On the other hand, there is 6.35 % decrease in the value of $Nu(Gr_{x^*})^{-1/4}$ at the same stream wise location, in the range of $0 \le M \le 1.0$



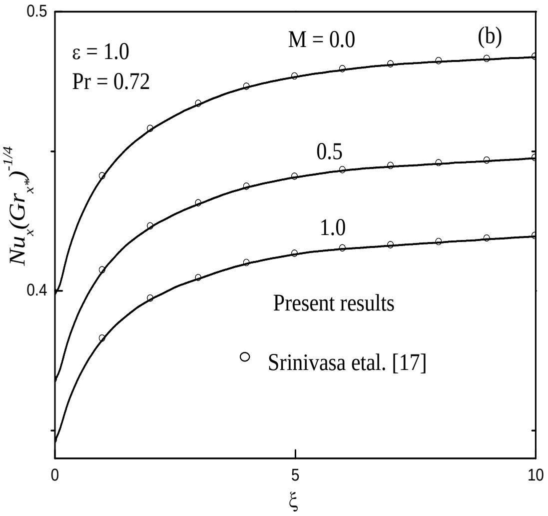


Figure: 2 (a) Skin friction (b) Heat transfer coefficient for different values of M

Figure 3 shows the velocity and temperature profiles for different value of M ($0 \le M \le 1.0$) with ε = 1.0 for Pr = 0.7 at the streamewise coordinate ξ = 5.0. It is observed that velocity decreases and temperature increases with increase of magnetic field (M). Indeed, the magnetic field normal to the flow in an electrically conducting fluid introduces a Lorentz force, which retards the flow. Consequently, the peak velocity decreases [See Fig.3 (a)] and the temperature increases [See Fig.3 (b)], within the boundary layer, due to retarding effect.

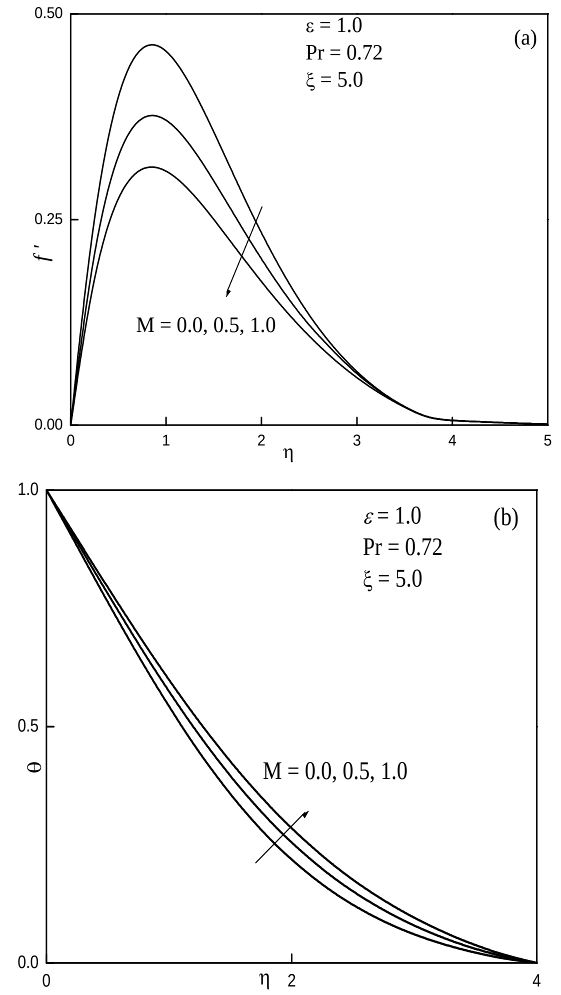


Figure: 3(a) Velocity and Temperature profile for different magnetic field M

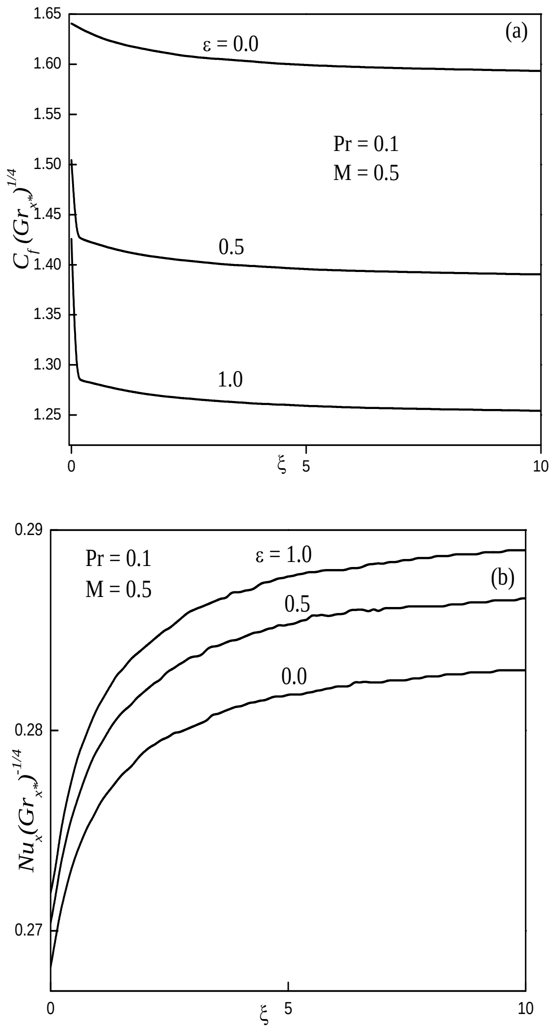


Figure: 4 (a) Skin friction and 4 (b) Heat transfer coefficient for different values of viscosity variation parameter ε

The effect of ε = (0.0, 0.5, 1.0) on the surface shear stress in terms of the local skin friction coefficient $[C_f(Gr_{x*})^{1/4}]$ and the rate of heat transfer in terms of the local Nusselt number $[Nu(Gr_{x*})^{-1/4}]$ are depicted graphically in figure 4(a) and 4(b), when M = 0.5 and Pr = 0.72. From this figure it can be noted that an increase in the variable viscosity variation parameter ε, the skin friction coefficient decreases and to increase the heat transfer rates. Here it is concluded that for high viscous fluids, the skin friction is less and the corresponding rate of heat transfer is high. It also seen that the skin friction decreases by 34 % and rate of heat transfer increases by 0.59 % as ε increases from 0.0 to 1.0 at the stream wise coordinate ξ = 5.0.

The velocity and temperature profile for variation viscosity parameter (ε) for an magnetic field (M = 0.5) and Prandtl number (Pr = 0.72) along streamewise coordinates is shown in figure 5(a) and 5(b) respectively. It is evident from figure that velocity increases while, temperature decreases with the increase of ε.

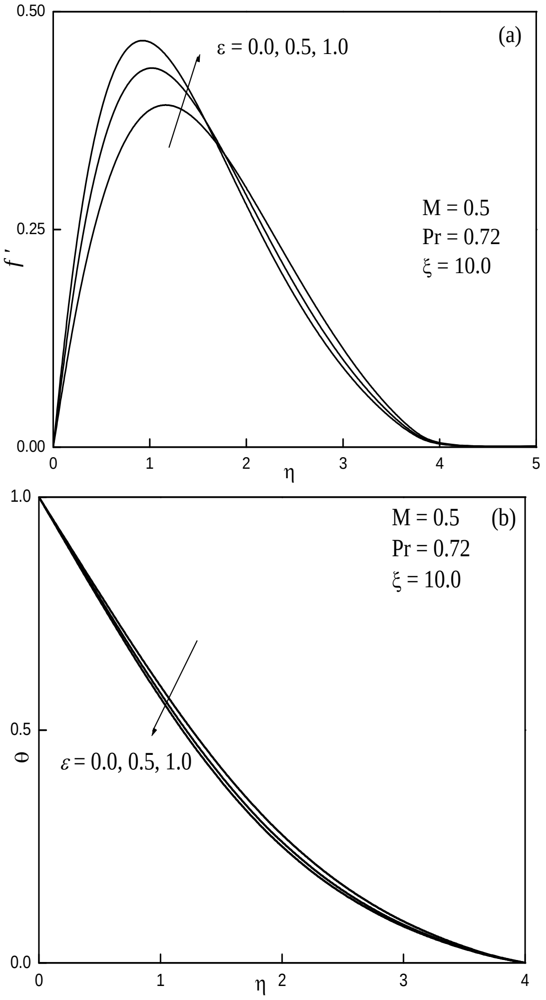


Figure: 5(a) Velocity and Temperature profile for different values of viscosity variation parameter ε

The influence of heat generation (Q >0) or absorption parameter (Q <0) on heat transfer coefficient [$Nu(Gr_{x*})^{-1/4}$] in the presence of the magnetic field (M = 0.5) is displayed in Fig.6. It is observed that $Nu(Gr_{x*})^{-1/4}$ decreases with the increase of Q (0 ≤Q ≤0.5) for irrespective of heat generation or absorption. On the other hand, there is a mild increase in $Nu(Gr_{x*})^{-1/4}$ during both heat generation Q > 0and heat absorption Q <0 Indeed, the percentage of decrease of $Nu(Gr_{x*})^{-1/4}$ when Q increases from Q =0.0 to Q =0.5 at ξ = 3.0 is 44.39% while the percentage of increase of $Nu(Gr_{x*})^{-1/4}$ when Q decreases from Q = 0.0 to Q =0.5 at ξ = 3.0 is 27.58% [Fig.6(a) & 6(b)]. Further, it is found that the direction of heat transfer gets reversed when Q=0.5 [Fig.6 (a)]. This is attributed to the fact that heat generation mechanism creates a layer of hot fluid near the surface and finally resultant temperature of the fluid exceeds the surface temperature resulting in the decrease of rate of heat transfer from the surface. The heat generation or absorption parameter does not cause any significant effect on skin friction coefficient [$C_f (Gr_{x*})^{1/4}$] and hence it is not shown here.

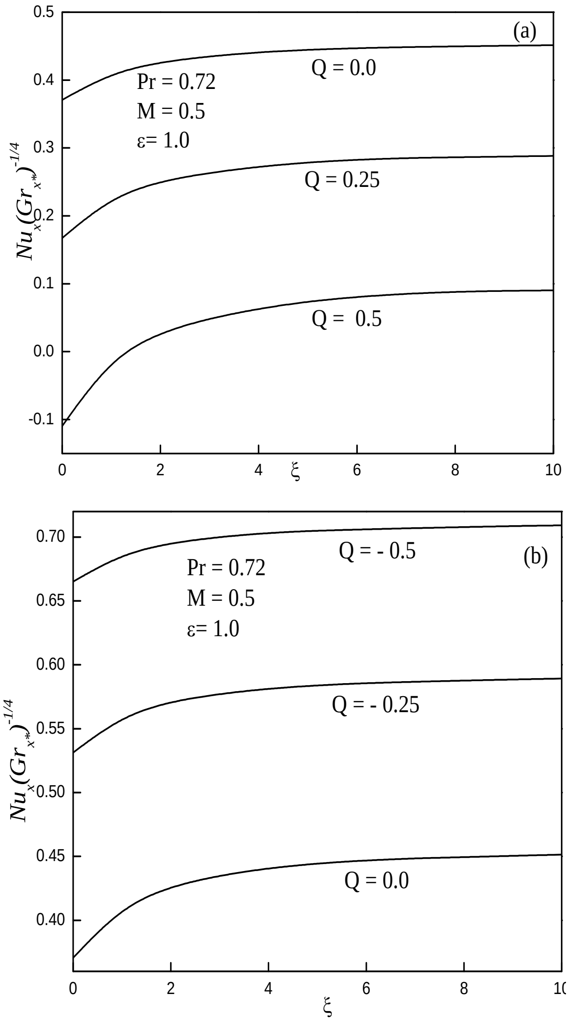


Figure: 6 Effect of (a) heat generation and (b) heat absorption parameter Q on heat transfer Coefficients

Fig. 7 depicts the effect of heat generation or absorption parameter on temperature profile in the presence of magnetic field (M = 0.5) with variable viscosity (ε). It is clearly observed that the thermal boundary layer thickness is increased in the presence of both heat generation and absorption. Further, it is evident from these figures that the present numerical results confirm to satisfy the thermal boundary layer conditions. The velocity profiles are unaffected by heat generation or absorption parameter and hence they are not shown here, for the sake of brevity.

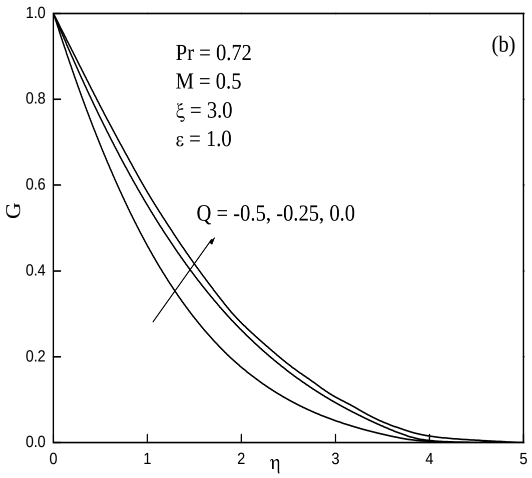


Fig: 7 Temperature profiles for (a) heat generation *Q* > 0and (b) heat absorption *Q* < 0

IV. CONCLUSIONS

For different values of pertinent physical parameters, the effect of temperature dependent viscosity on the natural convection flow of a viscous incompressible fluid along isothermal truncated cone has been studied. From the present investigation the following conclusions may be drawn:

(i) Both skin friction and heat transfer coefficients show a decreasing trend with the increase of magnetic parameter. However, skin friction is found to decrease and heat transfer increases near the leading edge.
(ii) In the free convection regime, the skin friction coefficient decreases and heat transfer coefficient increases with the increase of dimensionless distance. The velocity increases and temperature decreases along the conical surface with the increase of magnetic parameter.
(iii) The effect of increasing viscosity variation parameter results in decreasing the skin friction coefficient and increasing of the heat transfer coefficient.
(iv) The increase in the heat generation/absorption parameter results in decreasing the heat transfer coefficient. Also, during heat generation,the direction of heat transfer gets reversed.
(v) It is observed that the thermal boundary layer thickness is increased in the presence of both heat generation and absorption.

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